IPP-QM-10: The Bell–CHSH inequalities and possible responses

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MT25

The course

1. Basic quantum formalism
2. Density operators and entanglement
3. Decoherence
4. The measurement problem
5. Dynamical collapse theories
6. Bohmian mechanics
7. Everettian structure
8. Everettian probability
9. EPR and Bell's theorem
10. The Bell–CHSH inequalities and possible responses
11. Contextuality
12. The PBR theorem
13. Quantum logic
14. QBism
15. Pragmatism and relational quantum mechanics
16. Wavefunction realism

Today

Deriving the Bell–CHSH inequalities

Ways out

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Deriving the Bell–CHSH inequalities

Ways out

Deriving the Bell–CHSH inequalities

Today, we're going to look at the derivation of the Bell–CHSH inequalities, *à la* Bell (1976). This is the bread-and-butter of modern discussions on this topic.

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I won't discuss (3) any further, but I'll explain (1) and (2) now.

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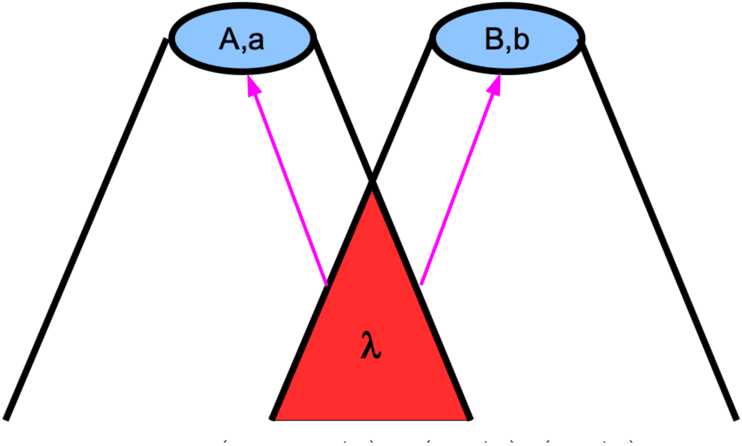
- These may include both observables and hidden variables, where nothing is said about what specifically λ contains.
- The condition of *local causality* states that if there is a correlation between two spacelike regions, then this must be causally explained by things in the λ of those two spacelike regions.

Local causality



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2. *Temporal asymmetry of causality:* the cause of any event lies in its temporal past, and not in its temporal future.
3. *Reichenbach's common cause principle*: if there are two correlated variables which do not have direct causal links then there is common cause of their correlations.

Factorisability

Bell (1976) argued that a key *consequence* (but not formulation) of local causality is *factorisability*:

$$\mathrm{Pr}(A, B|a, b, \lambda) = \mathrm{Pr}(A|a, \lambda)\mathrm{Pr}(B|b, \lambda),$$

where $\mathrm{Pr}(A, B|a, b, \lambda)$ is the joint probability in the theory for outcome A in region 1 associated with setting the variable a (this could be the direction of a spin meter) and outcome B in region 2 associated with setting the variable b, conditional on λ as specified before.

Measurement independence

Measurement independence states that the choice of variables a and b is independent of those things in λ such that the following hold:

$$\Pr(a|\lambda) = \Pr(a)$$
$$\Pr(b|\lambda) = \Pr(b)$$

Combining measurement independence and local causality

Combining measurement independence and local causality, we have

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So then using standard probability theory we have

$$\begin{aligned}\Pr(a, b) &= \sum_{\lambda} \Pr(a, b|\lambda)\Pr(\lambda) \\ &= \Pr(a)\Pr(b)\sum_{\lambda} \Pr(\lambda) \\ &= \Pr(a)\Pr(b).\end{aligned}$$

More manipulations

Next, one considers the following straightforward conditionalisation in probability theory:

$$\begin{aligned}
\Pr(A, a, B, b, \lambda) &= \Pr(A, a, B, b|\lambda)\Pr(\lambda) \\
&= \Pr(A, a|\lambda)\Pr(B, b|\lambda)\Pr(\lambda) \quad \text{(LC)} \\
&= \Pr(A|a, \lambda)\Pr(a|\lambda)\Pr(B|b, \lambda)\Pr(b|\lambda)\Pr(\lambda) \quad \text{(PT)} \\
&= \Pr(A|a, \lambda)\Pr(a)\Pr(B|b, \lambda)\Pr(b)\Pr(\lambda) \quad \text{(MI)}
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\end{aligned}
$$

Now take the marginal distribution by summing over λ and plug in the expression just derived:

$$
\begin{aligned}
\Pr(A, a, B, b) &= \sum_{\lambda} \Pr(A, a, B, b, \lambda) \\
&= \sum_{\lambda} \Pr(A|a, \lambda)\Pr(a)\Pr(B|b, \lambda)\Pr(b)\Pr(\lambda)
\end{aligned}
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More manipulations

We also have

$$\begin{aligned}\Pr(A, a, B, b) &= \Pr(A, B|a, b)\Pr(a, b)\\ &= \Pr(A, B|a, b)\Pr(a)\Pr(b) \quad \text{(MI)}\end{aligned}$$

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Equating this with the result on the previous slide gives

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$$\Pr(A, B|a, b) = \sum_{\lambda} \Pr(A|a, \lambda)\Pr(B|b, \lambda)\Pr(\lambda)$$

(To repeat: this is a result derived from just LC, MI, and probability theory.)

Deriving the Bell–CHSH inequality

Count a red light flashing as $+1$; count a green light flashing as -1. Then we can introduce the following expectation value:

$$\begin{aligned} E(A, B|a, b) = \Pr(A_R, B_R|a, b) - \Pr(A_G, B_R|a, b) \\ - \Pr(A_R, B_G|a, b) + \Pr(A_G, B_G|a, b). \end{aligned}$$

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We also have the following ingredients:

$$\Pr(A,B|a,b) = \sum_{\lambda} \Pr(A|a,\lambda)\Pr(B|b,\lambda)\Pr(\lambda)$$
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Combining these will give (**exercise:** derive it!):

$$E(A,B|a,b) = \sum_{\lambda} E(A|a,\lambda)E(B|b,\lambda)\mathrm{Pr}(\lambda)$$

Deriving the Bell–CHSH inequality

Now consider the *CHSH expression*, which considers the expectation values associated with different possible measurement settings:

$$CHSH := E(A,B|a,b) + E(A,B|a',b) + E(A,B|a,b') - E(A,B|a',b')$$

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Next we consider magnitudes. Recalling that

$$\left|\sum_\lambda F(\lambda)\Pr(\lambda)\right| \leq \sum_\lambda |F(\lambda)| \Pr(\lambda),$$

we have

$$|CHSH| \leq \sum_\lambda \Big| E(A|a,\lambda)\left\{E(B|b,\lambda)+E(B|b',\lambda)\right\}$$
$$+ E(A|a',\lambda)\left\{E(B|b,\lambda)-E(B|b',\lambda)\right\}\Big| \Pr(\lambda)$$

Deriving the Bell–CHSH inequality

If we also use

$$|FG| = |F|\,|G|\,,$$

$$\sum_\lambda \left(|F(\lambda) + G(\lambda)|\right) \Pr(\lambda) \leq \sum_\lambda \left(|F(\lambda)| + |G(\lambda)|\right) \Pr(\lambda),$$

we derive

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Now recalling that if $-1 \leq X \leq 1$ and $-1 \leq Y \leq 1$ then $0 \leq |X+Y| + |X-Y| \leq 2$, we have

$$\left|E(A,B|a,b) + E(A,B|a',b) + E(A,B|a,b') - E(A,B|a',b')\right| \leq 2$$

This is the Bell–CHSH inequality, which we can test experimentally!

Quantum theory violates the Bell–CHSH inequality

One can derive (see e.g. Redhead (1987) for the details) that in quantum theory,

$$\begin{aligned} &E(A, B|a, b) + E(A, B|a', b) + E(A, B|a, b') - E(A, B|a', b') \\ &\quad = 1 + 2\cos(2\phi) - \cos(4\phi) \end{aligned}$$

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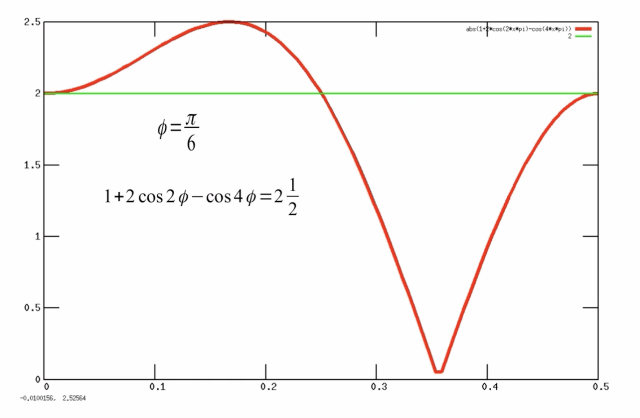

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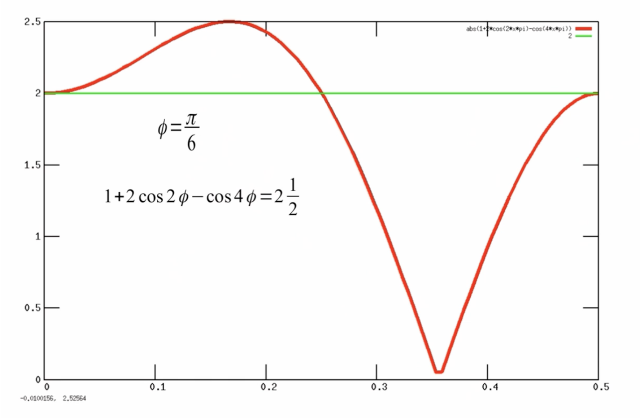

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(The bound on quantum violations of the Bell–CHSH inequality is called the *Tsirelson bound*.)

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  - Local causality
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- Assumptions used in this derivation:
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- Assumptions *not* used in this derivation:
  - Determinism
  - Perfect correlations
  - Hidden variables
  - Quantum theory

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- Not simply a relabelling of quantum theory using X-language.

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- Let's take a bit of time here to consider how Everett understands an EPR-Bell experiment without violating locality. (I largely follow Brown and Timpson (2016); see also Wallace (2012, ch. 8).)

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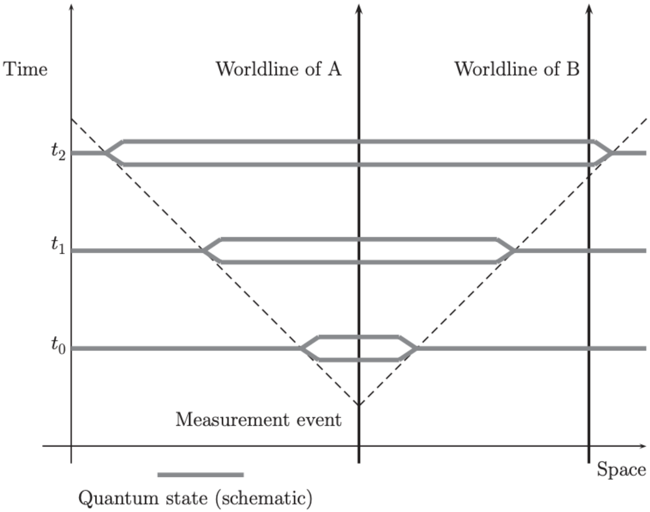
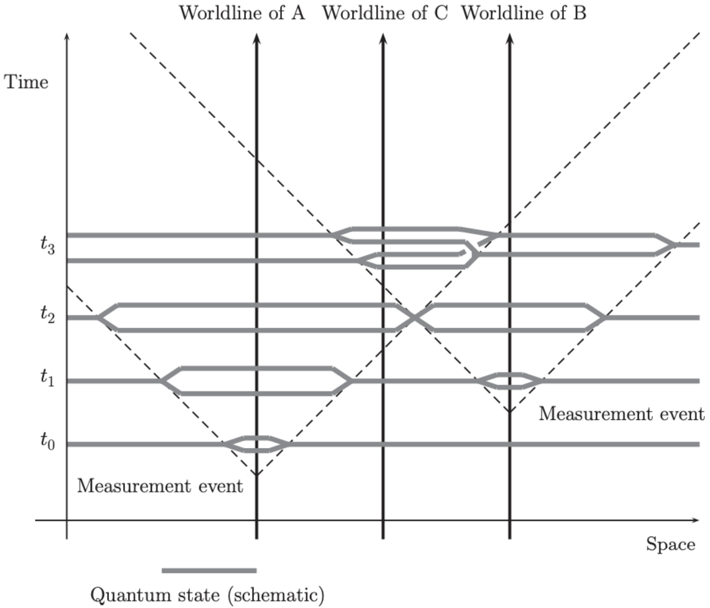
- However, in neither of those worlds at *A* will there be a definite outcome at *B*; relative to *A*, the electron and measuring device in *B* remain entangled.
- This is because the dynamics of measurement and branching at *A* are *entirely local* (since decoherence effects travels at the speed of light) and so for the measuring device and electron at *B* which are at a space-like separation from *A*, these effects will not yet have reached *B*.

Localised branching on Everett



Localised branching and interactions on Everett



No common cause principle needed in Everett!

- For the outcomes of the measuring device at A and B to be compared and for there to be a definite outcome on A's spin meter relative to a definite outcome on B's spin meter the two spin-meters (and electrons) need to be brought together and a joint measurement performed.

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- Only once the future light-cones of A and B cross will there be a definite outcome of one device relative to a definite outcome of the other and only then will it make sense to talk about correlations between spin-meter readings.

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- Only once the future light-cones of A and B cross will there be a definite outcome of one device relative to a definite outcome of the other and only then will it make sense to talk about correlations between spin-meter readings.
- Thus, on the Everettian account, *correlations arise not due to Reichenbachian common causes but rather due to local dynamics acting on an initially entangled non-separable state*.

Everettian lessons for Bell's theorem

- Thus, according to the Everettian, Bell, in relying on an assumption of Reichenbach's common cause principle to derive his local causality condition (LC) and subsequently his inequality, went beyond assuming locality, which is in fact captured entirely by (1) (Lorentz invariance) and (2) (temporal asymmetry of causality).

Everettian lessons for Bell's theorem

- Thus, according to the Everettian, Bell, in relying on an assumption of Reichenbach's common cause principle to derive his local causality condition (LC) and subsequently his inequality, went beyond assuming locality, which is in fact captured entirely by (1) (Lorentz invariance) and (2) (temporal asymmetry of causality).
- Hence, local causality (LC) and the condition of factorizability can be violated without a violation of locality.

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(NB: The terminology of ‘parameter independence’ and ‘outcome independence’ is from Shimony (1986, 1990).)

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- Signaling at the level of the ontic states.
- No-signaling emerges after statistical averaging.
- Relativistic invariance at the statistical level.
- E.g., Bohmian mechanics.

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- No-signaling built in at the level of ontic states.
- Spontaneous remote correlations.
- Still violates local causality.
- E.g., GRW collapse.
- (Also apparent violation of OI in Everett.)

Different views on parameter dependence and outcome dependence

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   - PD is action-at-a-distance
   - OD is "passion-at-a-distance" (Shimony) and so (claim) less bothersome for relativity.
3. "PD and OD are both just violations of local causality" (Maudlin (2014), who doesn't like splitting LC up—cf. his "fallacy of the unnecessary adjective".)

Ways out?

There are a few moves which someone wishing to avoid the Bell–CHSH theorem might make:

1. Argue that local causality can be violated without violating locality
   1.1 New notion of ‘quantum causality’?
   1.2 Everett
2. Argue that the factorisability condition can be decomposed into two components: ‘outcome independence’ and ‘parameter independence’ and that one (or both?) can be violated.
   2.1 Bohmian mechanics
   2.2 GRW
3. Deny measurement independence.
   3.1 Superdeterminism
   3.2 Retrocausality

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- Recall that the idea of measurement independence is that the experimenter can choose settings etc. in a way which is free from the influence of λ: $\Pr(a|\lambda) = \Pr(a)$, etc.

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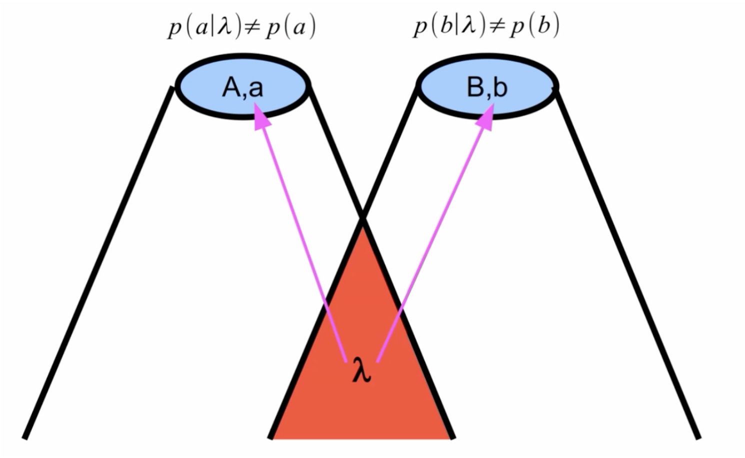
- (For more on superdeterminism, see a nice debate between Palmer (for) and Timpson (against) on the Oxford Philosophy of Physics YouTube channel.)

Superdeterminism



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- The causal influence propagates backwards in time.
- Need to avoid causal loops!

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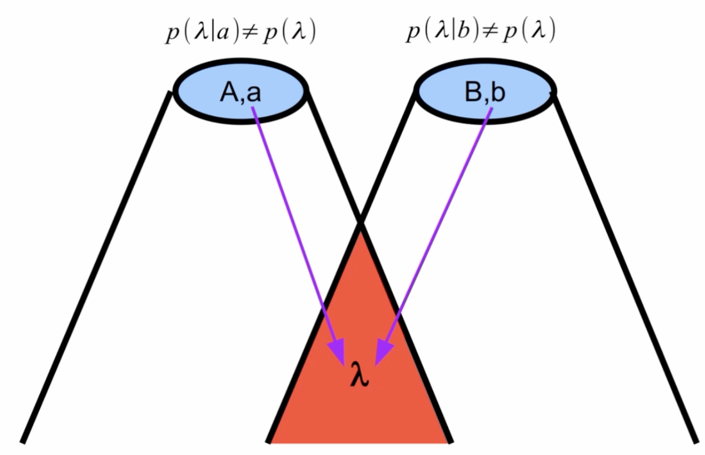

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Next week: the BKS and PBR theorems, which are other no-go theorems in the foundations of QM!

References I

- Jonathan Barrett, Robin Lorenz and Ognyan Oreshkov, "Quantum Causal Models", 2020.
- John S. Bell, "The Theory of Local Beables", Epistemological Letters, 1976.
- Harvey R. Brown and Christopher G. Timpson, "Bell on Bell's Theorem: The Changing Face of Nonlocality", in M. Bell and S. Gao (eds.), *Quantum Nonlocality and Reality: 50 Years of Bell's Theorem*, Cambridge: Cambridge University Press, 2016.
- J. P. Jarrett, "On the Physical Significance of the Locality Conditions in the Bell Arguments", Noûs 18, pp. 569–89, 1984.
- Tim Maudlin, "What Bell Did", Journal of Physics A: Mathematical and Theoretical 47, 424010, 2014.

References II

- Wayne Myrvold, Marco Genovese and Abner Shimony, “Bell’s Theorem”, in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, 2024.
- Michael Redhead, *Incompleteness, Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics*, Oxford: Clarendon Press, 1987.
- Abner Shimony, “Events and Processes in the Quantum World,” in R. Penrose and C. J. Isham (eds.), *Quantum Concepts in Space and Time*, Oxford: Oxford University Press, pp. 182–203, 1986.
- Abner Shimony, “An Exposition of Bell’s Theorem”, in A. Miller (ed.), *Sixty-Two Years of Uncertainty*, New York: Plenum, pp. 33–43, 1990.
- David Wallace, *The Emergent Multiverse*, Oxford: Oxford University Press, 2012.